DEVELOPMENT-SCHEME SELECTION FOR RIVER VALLEYS

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Abstract - This paper presents a new approach to the problem of selecting a development scheme for a river valley from preliminary surveys on candidate sites. Rather than proceeding by elimination, this approach identifies the scheme which minimizes the investment and operating costs. This minimization problem is decomposed using the Benders technique into a master problem covering the site selection and sizing aspects and a subproblem covering the production aspect. The former is solved by mixed-integer linear programming whereas the latter is transformed by a Lagrangian relaxation into a nonlinear network flow problem which is solved by a variant of the Frank-Wolfe method. Numerical results are reported.

1. INTRODUCTION

A virgin river valley can contain literally dozens of possible sites for the construction of a hydroelectric complex while each site, in turn, can offer a host of possibilities with respect to the type and size of the installations built. The result is that an almost infinite number of potential development schemes exists for each valley. The purpose of the work described in this paper was to find a method for identifying the best.

The detailed study of a hydroelectric development scheme comprises several stages. The first involves topological surveys of the land upstream of each site to determine the locations where retaining dikes will be required to hold back the water for the reservoir. Meanwhile, at the site itself, surface and underground geological studies are required to determine the type, strength and stability of the ground on which the dam and power station will be built. Hydroelectric surveys are also undertaken at each site to establish the characteristics of the water flow. Once the various surveys have been completed, of course, the data has to be analyzed and validated while hydraulic and other studies must be carried out before a specific project can be drawn up for the site selected. The final stage in this process is the cost evaluation.

Such a study is obviously time-consuming, strenuous and costly and can consequently not be undertaken blindly for any development scheme. Jack Fritz [1] recently observed that "the budget required to perform all site studies for a small hydropower system could reach 25 percent of total project cost", which is a hefty percentage. It is because of these high costs, in fact, that utilities throughout the world tend to perform a reconnaissance study based on summary surveys in order to rapidly eliminate the least interesting sites.

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The danger is that a site can be eliminated without taking other sites into account. Individual sites may not appear economically viable in themselves but when considered as part of an entire development scheme they can be valuable assets. For example, the construction of a hydroelectric complex including a seasonal reservoir at a given site may seem nonviable, economically, if one considers no more than the production at that complex alone but, if the overall plan is to build generating stations downstream of this complex whose production will be substantially increased by the inclusion of the seasonal storage reservoir, then the site is worth developing. In view of this, it is not sites that should be eliminated in the reconnaissance studies as much as development schemes but, as mentioned earlier, the fact that there are infinitely more schemes than sites, the exercise could be both long and costly. One way of solving this dilemma is to replace the elimination-based approach by the following optimization approach:

1) First, estimate the development cost for each site, based on the preliminary surveys, as a function of the reservoir capacity and power plant capacity. (It is assumed that this estimate will not exceed the real cost). Then fix ITER = 1.
2) Determine the development scheme which minimizes the combined investment and operating costs using the method described below. If ITER = 1, go directly to step 4.
3) If the development scheme just identified is identical to the one found at the previous iteration, stop the search because the scheme is an optimal one. If not, go to the next step.
4) Make a detailed study of the last development scheme in order to determine the exact cost. If this is different from the estimate, replace the latter by the exact cost, make ITER = ITER + 1 and return to step 2.

The development scheme determined according to the above procedure is really optimal, for three reasons: 1) it minimizes the investment and operating costs; 2) its cost is exact; 3) because of the assumption made in step 1, the cost of the other schemes is not overestimated. As for the number of iterations required and, consequently, the number of development schemes to be studied in detail, these depend on the original estimate of the investment cost. If this was exact, a simple iteration is required whereas if not, several iterations may be needed. It should not be forgotten, however, that even if the development scheme changes at each iteration, the sites involved do not necessarily change. Only the size of some of the installations might be altered.

Consequently the number of sites to be studied in detail is not forcibly higher with this method than in the elimination approach.

The remainder of this paper is devoted to describing the optimization method (required in step 2 in the foregoing algorithm) used to determine the development scheme which would minimize the combined investment and operating costs. Some numerical results are given in section 4.

2. PROBLEM FORMULATION

The optimum development scheme for a valley was defined above as being that which minimizes the sum of the investment and operating costs. This calls for a means for determining the system operating cost once the production facilities have been built and connected to the network. There are two possible methods. The first, which can be qualified as the global approach, is to establish the operating policy for the entire system that would minimize the overall operating cost. The second, local approach assumes that the place chosen for the new production facilities under the load duration curve has already been determined by the decision maker; for example, it could be the cross-hatched area shown in Figure 1. The problem therefore consists in determining the operating policy for the planned facilities which will fulfill the energy requirements represented by the cross-hatched surface at the lowest operating cost.

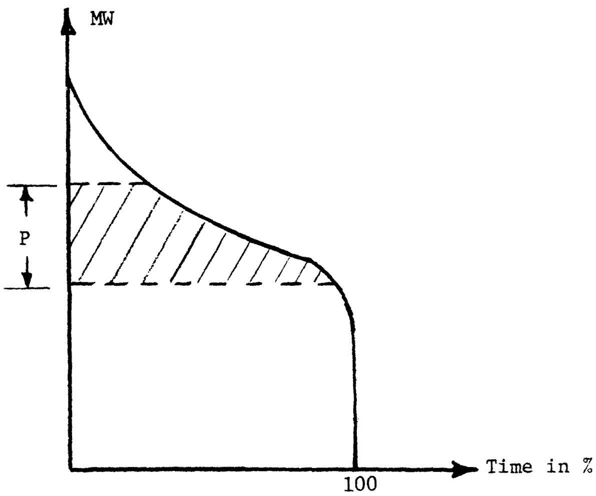

Fig. 1: Load-duration curve

Obviously the second method is far easier than the first, since the operating policy for existing installations does not have to be established but, on the other hand, it is not as good, since it fixes the production of the new installations before knowing what form they will take.

Despite this shortcoming, however, it is the second method which is usually adopted in practice, and will be followed here, because system operators and system planners are rarely the same people. The former decide what the new plant should produce, as illustrated in Figure 1, whereas the latter plan the system in terms of the decision made.

2.1 The Objective Function

According to Figure 1, the development scheme selected for the valley must have a generating capacity of P MW if the demand is to be satisfied. However, if the emphasis is put on the system having such a capacity, there is always a risk of ending up with a scheme that is not economically viable at all, or far less so than a smaller one. To avoid this, the program must be instructed that it can select a scheme with a lower capacity than P MW but the difference must be made up by an alternative energy source whose purchase cost is \$ cav/MW and operating cost is \$ cap/MWh. Depending on the location of the cross-hatched area below the load duration curve, this source can be nuclear, thermal, diesel units or gas turbines. In a word, the program will have a choice between building hydro plants, alternative-energy generating facilities, or a mixture of the two.

Obviously the optimum solution is the one which minimizes the investment and operating costs or, in mathematical terms, minimizes:

$$\sum_{i=1}^{n} \left(crf_i \cdot IR_i + crv_i \cdot HR_i + cpf_i \cdot IP_i + cpv_i \cdot PP_i \right)$$

$$+ cav \cdot PA + \sum_{k=1}^{K} cap \cdot EA_k \qquad (1)$$

where

IR_i	=	binary variable which is 1 if a reservoir is built at site i and 0 otherwise
crf_i	=	fixed cost of building a reservoir at site i (\$)
HR_i	=	maximum level of reservoir i (m)
crv_i	=	variable cost of reservoir construction at site i (\$/m)
IP_i	=	binary variable which is 1 if a power plant is built at site i and 0 if not
cpf_i	=	fixed cost of building a power plant at site i (\$)
PP_i	=	installed capacity at power plant i (MW)
cpv_i	=	variable cost of building a power plant at site i (\$/MW)
PA	=	installed capacity at the alternative source (MW)
cav	=	variable cost of building an alternative source (\$/MW)
EA_k	=	energy generated by the alternative source over a period k (MWh)
cap	=	production cost for alternative energy (\$/MWh)
K	=	number of periods considered

Note that in the case of criterion (1), the cost of reservoir i is assumed to increase linearly with its size, which may not be the case in practice. When a reservoir is expanded, new dikes are often required to prevent water from spilling into other valleys, thus occasioning new fixed costs as well as a higher variable cost. Furthermore it is far from certain, despite the indications of function (1), that the cost of an alternative generating station increases linearly with its capacity since economies of scale can be achieved. Finally, another very debatable assumption underlying criterion (1) is that the production of the alternative generating station over period k, EA_k, can be determined with certainty, k periods in advance. This production, it will be recalled, is equal to the demand minus the hydroelectricity produced, which in turn depends on the natural water inflow, the latter being impossible to predict very accurately a long time in advance. In other words, criterion (1) is far from perfect but, if the necessary data is available, it could be improved. Nevertheless it must be remembered that if it is replaced by another, nonlinear, function to take account of the economies of scale or the increase in the number of dikes, the master problem described in section 3.2 can no longer be resolved by linear programming. If, on top of this, it is decided to take account of the random nature of the inflow, then the operating subproblem has to be resolved stochastically rather than using the deterministic method presented below. It is partly in an aim to avoid techniques that are difficult to apply that criterion (1) was adopted from the outset, despite its shortcomings. At this point, there are plans in the air to change it but, even if these plans materialize, the approach presented here, which consists in decomposing the problem into a scheduling problem and an operating subproblem, would still be valid.

2.2 The Constraints

The minimization of function (1) must be done under a large number of constraints which can be grouped as follows:

- individual reservoir operation constraints
- drawdown and dam height constraints
- powerhouse location constraints
- production constraints

The operating constraints for reservoir i can be expressed mathematically as follows:

$$X_{ik} = X_{ik-1} + A_{ik} - U_{ik} - V_{ik} \tag{2}$$

$$X_{ik} \leq \bar{X}_i \tag{3}$$

$$U_{ik} \leq \bar{U}_i \tag{4}$$

$$X_{io} = X_{iK} = \bar{X}_i \tag{5}$$

where

$$A_{ik} = y_{ik} + \sum_{j \in \Gamma_i} (U_{jk} + V_{jk}) \tag{6}$$

Equation (2), the state equation for reservoir i, specifies that X_{ik}, the reservoir content at the beginning of period k, is equal to the content at the beginning of period k-1 plus A_{ik}, the total inflow in period k, less U_{ik}, the effective draft in period k, less V_{ik}, the spillage during period k. The total inflow A_{ik}, as specified in Eq. (6), is equal to y_{ik}, the natural inflow during period k, plus the sum of the effective discharges and spillages from the reservoirs immediately upstream. Constraint (3) shows that the content of reservoir i at the beginning of period k cannot exceed $\bar{X}_i$, the reservoir's storage capacity. Constraint (4) limits the effective draft from reservoir i during period k to the value $\bar{U}_i$, which represents the discharge capacity of the generating station. Constraint (5) fixes the content of reservoir i at the beginning and end of the period studied to the value $\bar{X}_i$.

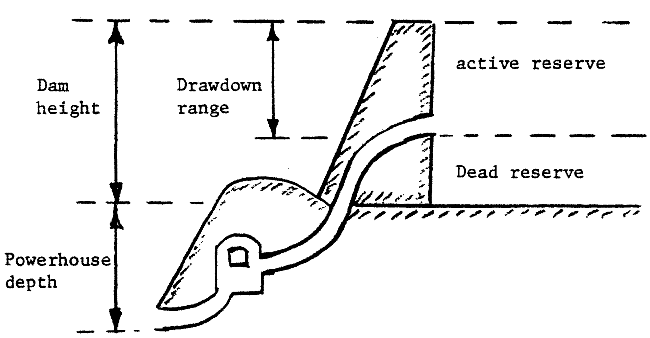

Fig. 2: Hydroelectric Scheme

As seen in Fig. 2, the useful reserve $\bar{X}_i$ depends on the height of the dam HR_i and the height of the water inlet HD_i. The water stored below the inlet, the so-called dead reserve, is denoted by XD_i. The head of water at powerhouse i is equal to the depth of the powerhouse HP_i plus the height of the water level in the reservoir. It is important that the head not vary too much, in order that the generating station can operate at more or less maximum capacity. One means of doing this is to fix the drawdown range (which incidentally is equal to $HR_i - HD_i$) to a fraction (0.10 to 0.15) of the maximum head. This fraction, or drawndown coefficient, is represented by the letters ac in the constraint given below.

$$HR_i - HD_i \leq ac \cdot (HR_i + HP_i) \tag{7}$$

$$HD_i = H_i\ (XD_i) \tag{8}$$

$$HR_i = H_i\ (XD_i + \bar{X}_i) \tag{9}$$

$$HP_i \leq \overline{HP}_i \cdot IP_i \tag{10}$$

$$HR_i \leq \overline{HR}_i \ .\ IR_i \tag{11}$$

H_i in Eqs. (8) and (9) represents an increasing concave function which establishes the relationship between the reservoir content and the height of the water level above the foot of the dam. The symbol $\overline{HP}_i$ in Eq. (10) represents the maximum depth to which the generating station at site i can be located while $\overline{HR}_i$ in Eq. (11) denotes the maximum height the dam at site i can attain.

It should be emphasized that only the location of the dam is fixed a priori for dam i; for example, this location will be assumed to be at an altitude of $foot_i$ metres. As for generating station i, it could be located anywhere between altitudes "$foot_i - \overline{HP}_i$" and "$foot_i$". Therefore, denoting the altitude of generating station i as AP_i and the maximum height the water level in reservoir i can attain as AR_i, we see that

$$AR_i - AP_i = HR_i + HP_i \tag{12}$$

$$AR_i \geq foot_i \tag{13}$$

$$AP_i \geq foot_i - \overline{HP}_i \tag{14}$$

If sites i and i+1 are very close together, $foot_i - \overline{HP}_i$ could be lower than $foot_{i+1} + \overline{HR}_i$, with the result that the reservoir at site i+1 could flood the powerhouse at site i. To avoid this, the following constraint must be added:

$$AP_i \geq AR_{i+1} \tag{15}$$

In addition, care must be taken to ensure that the space between the crest of the dam and its foot is full, otherwise the project selected by the computer program risks comprising no more than a section of dam at maximum altitude, a powerhouse at mimimum altitude and nothing between the two (which incidentally would provide a high head at little cost!). This situation can also be avoided by adding four linear constraints to the model:

$$AR_i - HR_i \leq altm - (altm - foot_i) \cdot IR_i \tag{16}$$

$$AR_i - HR_i \geq foot_i \cdot IR_i \tag{17}$$

$$AP_i + HP_i \leq altm - (altm - foot_i) \cdot IP_i \tag{18}$$

$$AP_i + HP_i \geq foot_i \cdot IP_i \tag{19}$$

where $altm = foot_i + \overline{HP}_i$

The final constraints to be taken into consideration concern the electrical demand to be satisfied. First there is the maximum load P that the new facilities must be able to provide. Therefore

$$\sum_{i=1}^{n} PP_i + PA \geq P \tag{20}$$

where PP_i represents the peak power generated by power station i. Also to be satisfied is the electrical demand in period k, E_k. Consequently,

$$\sum_{i=1}^{n} EP_{ik} + EA_k \geq E_k \tag{21}$$

where EP_{ik} denotes the output from power station i in period k. Naturally the production of the alternative power station cannot exceed its capacity, with the result that

$$EA_k \leq m_k \cdot PA \tag{22}$$

where m_k represents the number of hours in period k.

Both the power and the energy generated at power station i are functions of the head and the amount of water flowing through the turbine. Several assumptions have been formulated for these functions in the past including:

$$PP_i = \alpha \cdot \left[HP_i + H_i\ (XD_i + 0.5\bar{X}_i)\right] \cdot \bar{U}_i \tag{23}$$

$$EP_{ik} = \alpha \cdot m_k \cdot \left[HP_i + H_i\ (XD_i + X_{ik-1})\right] \cdot U_{ik} \tag{24}$$

where α is a conversion factor. To calculate the head in Eq. (23), it is assumed that reservoir i will be half full at the time the peak occurs.

Before concluding this description of the problem, it should be mentioned that although not explicitly noted above, all the variables involved can only have non-negative values.

3. METHOD OF SOLUTION

The problem just described will be resolved by the Benders decomposition method. This consists in decomposing the original problem into two problems: a master and a subproblem, each being much easier to solve than the original. The master problem, in our case, will be to choose a development scheme for the river valley and it will be resolved by linear programming. The subproblem will be to determine the optimum operating policy for the installation forming part of the selected development scheme. The subproblem will be transformed into a nonlinear network flow problem, via a Langrangian relaxation, and once multipliers have been selected, the subproblem will be resolved by a variant of the Frank-Wolfe method [2, 3].

3.1 Generalized Benders Method

The Benders decomposition method, generalized by Geoffrion [4], is used for solving problems of the following form:

$$\operatorname*{Min}_{w,z} F(w,z) \text{ subject to } G(w,z) \leq 0;\ w \varepsilon W;\ z \varepsilon Z \tag{25}$$

where z is vector of complicating variables in the sense that (25) is a much easier problem in w when z is temporarily held fixed. Geoffrion quotes three cases where the Benders method can be used advantageously to solve problem (25).

(a) for fixed z, the problem separates into a number of independent optimization problems, each involving a different subvector of w;

(b) for fixed z, (25) assumes a well-known special structure for which efficient solution procedures are available;

(c) problem (25) is not a convex problem in w and z jointly, but fixing z renders it so in w.

Our problem falls into the second category in the sense that once the development scheme has been selected, and, therefore, the vector z fixed, the problem becomes an operating problem for which various methods of solution have been put forward in the past [5 - 13], in addition to that proposed below.

The key to Benders' decomposition is to project problem (25) into the z space, which yields the following problem:

$$\operatorname*{minimize}_{z} S(z) \text{ subject to } z \varepsilon Z \cap R \tag{26}$$

where

$$S(z) = \operatorname*{infimum}_{w} F(w,z) \text{ s.t. } G(w,z) \leq 0,\ w \varepsilon W \tag{27}$$

and

$$R = \{z : G(w,z) \leq 0 \text{ for some } w\} \tag{28}$$

Here, (26) is the master and (27) the subproblem. If, now, the constraint $G(w,z) \leq 0$ is added to the objective function with a multiplier λ, the subproblem becomes

$$S(z) = \operatorname*{supremum}_{\lambda \geq 0} \left[\text{infimum } F(w,z) + \lambda G(w,z)\right] \tag{29}$$

and the master can simply be rewritten

$$\operatorname*{mimimum}_{z \varepsilon Z \cap R} \left\{\operatorname*{supremum}_{\lambda \geq 0} \left[\operatorname*{infimum}_{w \varepsilon W} F(w,z) + \lambda G(w,z)\right]\right\} \tag{30}$$

or, if we use the definition of supremum as the lowest upper bound,

$$\operatorname*{minimize}_{z \varepsilon Z \cap R} q \tag{31}$$

subject to:

$$q \geq \operatorname*{infimum}_{w \varepsilon W} \{(F(w,z) + \lambda G(w,z)\} \quad \text{all } \lambda \geq 0 \tag{32}$$

Each of the constraints in (32) is called a Benders cut. In view of the very large number of such constraints, the most natural strategy for solving problem (31) - (32) is relaxation [14], which proceeds as follows: Begin by solving a relaxed version of (31) - (32) that ignores all but a few constraints of (32). If the resulting solution does not satisfy all of the ignored constraints, then generate and add to the relaxed problem one or more violated constraints and solve it again. Continue in this fashion until a relaxed problem satisfies all of the ignored constraints.

This is the strategy which served as inspiration for the following algorithm: Denote by z_p, q_p, w_p and λ_p the values of z, q, w and λ obtained at iteration p.

Step 1: Set p = 1, $q_p \leq -\infty$, UBD = ∞, and z_1 to any value of $z \varepsilon Z \cap R$. Go to step 2.

Step 2: Solve the subproblem:

$$\operatorname*{Mimimize}_{w \varepsilon W} F(w,z_p) \text{ subject to } G(w,z_p) \leq 0.$$

Let w_p be an optimal solution. If

$$q_p \geq F(w_p,z_p) - \varepsilon,$$

where ε is the desired precision, stop. Otherwise find λ_p, the value of λ corresponding to the solution (w_p, z_p), set UBD = $F(w_p,z_p)$, p = p+1, and go to step 3.

Step 3: Solve the master problem

$$\operatorname*{minimize}_{z \varepsilon Z \cap R} q$$

subject to:

$$q \geq \operatorname*{infimum}_{w \varepsilon W} \{F(w,z) + \lambda_i \cdot G(w,z)\};\ i = 1,2,\ldots,p-1 \tag{33}$$

Let (z_p, q_p) be the optimal solution. If

$$q_p \geq \text{UBD} - \varepsilon$$

stop. Otherwise go to step 2.

Two problems hamper the use of the algorithm just described. The first is to find the z's that form part of the set R or, which comes to the same thing, to ensure that the z_p selected in Step 3 will be such that a solution will be obtained to the subproblem in Step 2. When applied to the problem of section 2, this amounts to wondering how to be sure the development scheme selected by the master will satisfy the energy demand, i.e. constraint (21), at all times. In fact the only way to do so is to solve the operating subproblem, which could be very time-consuming if there are several schemes under investigation. But there is a way of getting round this difficulty, which consists in allowing generation shedding if constraint (21) cannot be fulfilled, although at a cost so high that shedding will not in fact take place with the optimal scheme. Mathematically, this involves changing criterion (1) for the following criterion:

$$\sum_{i=1}^{n} (crf_i \cdot IR_i + crv_i \cdot HR_i + cpf_i \cdot IP_i + cpv_i \cdot PP_i) + cav \cdot PA + \sum_{k=1}^{K} (cap \cdot EA_k + cs \cdot ES_k) \qquad (34)$$

and constraint (21) for

$$\sum_{i=1}^{n} EP_{ik} + EA_k + ES_k \geq E_k \qquad (35)$$

where ES_k denotes the energy shed during period k and cs the cost of the shedding operation in \$/MWh. With this new model, there will always be a solution to the subproblem in step 2 whatever the development scheme selected in step 3. In other words, all $z \varepsilon Z$ is also comprised in R.

The second problem involved in using the above algorithm is that constraints (33) must be evaluated, which means solving p-1 minimization problems at iteration p for each value of z considered. For the problem in section 2, for instance, this amounts to solving p-1 operating problems for each development scheme under study. In view of the possible enormity of such a table and the extensive CPU time involved, it seemed wiser to replace (33) by the following heuristic cuts:

$$q \geq F(w_i,z) + \lambda_i G(w_i,z); \; i = 1,2,\ldots,p-1 \qquad (36)$$

where w_i denotes the solution to the subproblem found at iteration i. However, it is important to remember that the right-hand side of (36) is bigger than or equal to that of (33), with the result that the value of q_p obtained with this constraint could be higher than that obtained with (33). Moreover, if z_p is very different from z_i, $F(w_i,z) + \lambda_i G(w_i,z)$ could be a poor approximation of the exact cut, with the risk that the solution selected may not be the right one. To prevent this, the old cuts, i.e. those obtained with a z very different from z_p, should be cast aside and only the most recent ones applied.

3.2 The Master Problem

In addition to constraints (36), the master includes constraints (7) to (20) and (23). As mentioned earlier, the problem is solved by linear programming with mixed integer variables (MPSX) but only after constraints (8), (9) and (23) have been piecewise-linearized.

To linearize constraint (23) for example, two new variables, $DUM1_i$ and $DUM2_i$ were introduced where

$$DUM_1 \triangleq \alpha \cdot [HP_i + H_i (XD_i + 0.5\bar{X}_i)] + \bar{U}_i$$

and

$$DUM_2 \triangleq \alpha \cdot [HP_i + H_i (XD_i + 0.5\bar{X}_i)] - \bar{U}_i$$

with the result that (23) can be rewritten quite simply as:

$$PP_i = \tfrac{1}{4} (DUM1_1^2 - DUM2_2^2)$$

The terms $DUM1_i^2$ and $DUM2_i^2$ were then piecewise-linearized.

3.3 The Subproblem

The operating subproblem consists in determining the values of variables U_{ik}, EA_k and ES_k i=1,...,n, k=1, ...,K which

$$\text{minimize} \sum_{k=1}^{K} (cap \cdot EA_k + cs \cdot ES_k) \qquad (37)$$

under constraints (2) - (6), (22) and (35). Let us denote by W_{ik} the set of U_{ik} which satisfy constraints (2) - (6) and by T_k the set of EA_k which satisfy constraint (22). Let us then add constraint (35) to criterion (37) with a multiplier γ_k. The subproblem can then be rewritten

$$\max_{\substack{\gamma_k \\ k=1,\ldots,K}} \Big\{ \inf_{\substack{U_{ik} \varepsilon W_{ik} \\ \forall i,k}} \sum_{k=1}^{K} \sum_{i=1}^{n} \gamma_k EP_{ik} + \inf_{\substack{ES_k,\, EA_k \varepsilon T_k \\ \forall k}} \sum_{k=1}^{K} [(cap + \gamma_k)EA_k + (cs + \gamma_k) \cdot ES_k - \gamma_k E_k] \Big\}$$

and the latter solved by proceeding as follows:

Step 0: Assign an initial value to γ_k, k=1,...,k

Step 1: Solve the network flow problem

$$\text{minimize} \sum_{k=1}^{K} \sum_{i=1}^{n} \gamma_k EP_{ik}$$

under constraints (2) - (6) and with EP_{ik} as defined in (24).

Step 2: Set $DEF_k = E_k - \sum_{i=1}^{n} EP_{ik}$, and let

$$\gamma_k^* = \begin{cases} 0 & \text{if} \quad DEF_k \leq 0 \\ -\,cap & \text{if} \quad 0 < DEF_k \leq m_k PA \\ -\,cs & \text{if} \quad DEF_k > m_k PA \end{cases}$$

If $\gamma_k = \gamma_k^*$ for all k, stop. Otherwise, set $\gamma_k = \gamma_k^*$, k=1,...,K, and go back to step 1.

Although this method has yielded good results for the example described in section 4, we are very aware that it could cause instability because the solution to the network flow problem could change radically from one iteration to the other. It is true that this can be avoided by limiting the changes in the solution from one iteration to the other, but there is no guarantee then that the solution will converge. For this reason, it may be preferable to resort to a primal type of method to resolve the subproblem instead of the dual method presented above.

4. NUMERICAL EXAMPLE

The method just described has been tested on Hydro-Québec data related to the possible development of four hydro plants on the Romaine River on the North Shore of the St. Lawrence in eastern Québec. Table 1 gives the various possibilities with regard to reservoir height and plant at each of the sites considered.

Table 1: Possible Sites

Site	Elevation of dam (ft)	Maximum height of reservoir (ft)	Maximum depth of powerhouse (ft)
1	170	120	100
2	460	190	190
3	900	750	165
4	1260	340	110

Despite the fact that this example does not have a very wide range of sites, it does offer, as clearly visible from Table 1, an unlimited number of possibilities with respect to the size of the installations. The cost of such installations in terms of their size is given in Table 2 below:

Table 2: Cost of Installations

Site	Installation	Fixed cost $\$ \times 10^6$	Unit cost $
1	reservoir	28.4265	91818./ft
	central	31.7034	129./kW
2	reservoir	44.2067	120331./ft
	central	84.5071	80./kW
3	reservoir	26.4714	122334./ft
	central	47.5412	94./kW
4	reservoir	38.8507	192597./ft
	central	38.9974	90./kW

The construction cost of the alternative power plant was fixed at $131/kW and its operating cost at $25/MWh. The cost of the generation-shedding energy was fixed at $400/MWh.

The relationship between the water content and reservoir level is given in Table 3, the drawdown coefficient having been fixed at 0.10 for each reservoir.

Table 3: Reservoir Content vs Water Level

Site 1		Site 2		Site 3		Site 4	
Height (ft)	Contents (Gft^3)	Height (ft)	Contents (Gft^3)	Height (ft)	Contents (Gft^3)	Height (ft)	Contents (Gft^3)
50	0.72	40	0.43	150	15.66	140	5.4
75	2.00	90	4.94	300	60.45	240	78.76
100	3.52	140	16.80	450	132.6	290	195.94
120	5.99	190	36.51	550	219.47	340	495.47
				600	316.91		
				650	489.69		
				700	858.27		
				750	1608.8		

The data presented in Table 4 include the electrical demand, E_k, to be met by the new installations every month.

Table 4: Electrical Demand and Mean Inflow in the Valley for Each Month of the Year

Month	Demand (GWh)	Inflow (Gft^3)	Month	Demand (GWh)	Inflow (Gft^3)
1	766.3	12.79	7	597.6	86.31
2	936.0	12.79	8	543.1	28.77
3	1056.5	9.59	9	504.0	15.99
4	997.0	6.39	10	565.4	19.18
5	813.1	6.39	11	580.3	35.16
6	729.1	60.74	12	669.6	25.57

The peak output, denoted by P in constraint (20), was fixed at 1420 MW. The table also shows the monthly mean total inflow (in Gft^3) into the Romaine River. To determine the natural flow at each site, the total inflow is simply multipied by 0.07 for site 1, 0.14 for site 2, 0.13 for site 3 and 0.66 for site 4.

The operating subproblem was resolved for a one-year period only, using the mean inflow values and demand figures given in Table 4, with the result that the reservoir could very well be underdimensioned. Actually, to evaluate the reservoir size properly, the operating subproblem would have to be solved for many years, including periods of heavy and light inflow, but unfortunately the data required for this purpose are not available.

Finally, at the request of Hydro-Québec, a number of constraints were incorporated to limit the utilization factor of each power station (hydro alternative) to 0.8.

Determination of the optimal development scheme for this valley using the Benders method as described above took only 0.233 minutes of CPU time on the IBM-370 computers, most of this time being taken by the simplex to resolve the master problem. The master comprised 82 constraints (plus the cuts) and 197 variables including eight integers. The solution of this problem called for an average of 350 iterations of the simplex.

Resolution of the subproblem required an average of only four iterations of the algorithm presented in section 3.3 to generate an optimal solution, never more than nine.

The optimal development scheme was obtained after generating just four Benders cuts. As expected from section 3.1, the selection of a heuristic cut resulted in the value of the objective function of the master problem being higher than the real value (see Table 5). The optimal scheme obtained comprises three powerhouses and three reservoirs, as defined in Table 6.

Table 5: Lower and Upper Cost Limits Determined by the Resolution

Iteration	Lower cost limit ($ millions)	Upper cost limit ($ millions)	Real cost ($ millions)
1	- 119775.	2188.89	2188.69
2	479.44	2188.89	6097.68
3	488.92	703.76	703.76
4	753.81	-	-

Table 6: Optimal Solution

Site i	HR_i (ft)	$\bar{X}_i$ (Gft^3)	XD_i (Gft^3)	$\bar{U}_i$ (Gft^3)	HD_i (ft)	PP_i (MW)
1	120	2.59	3.4	33.4	100	208
2	190	14.19	22.32	31.1	170	329
3	750	1056.45	555.35	28.8	165	695
4	0	0	0	0	0	0

5. CONCLUSION

This paper has presented a new approach to the problem of selecting a development scheme for a valley based on preliminary surveys of potential sites. The conventional approach consisted in eliminating as many sites as possible on the basis of the preliminary surveys then carrying out further surveys at the sites selected as candidates in order to determine those worthwhile developing. The problem with this approach is that it can eliminate a site which in itself is devoid of interest but, with others, forms the most attractive scheme for the valley. This shortcoming is avoided in the new method since no sites are eliminated. By contrast, the approach is constructive. Based on preliminary survey data, it first determines the development scheme that minimizes the sum of the investment and operating costs. This scheme is then studied in detail to determine the exact cost. If the latter is higher than the original estimate, a new optimization is performed with this cost to see whether a less expensive scheme exists. If so, the new scheme is studied in detail and, if necessary, a further iteration is undertaken.

This paper shows that to find the scheme which minimizes the sum of the investment and operating costs, a nonlinear optimization problem with mixed integer variables must be solved. The Benders decomposition method selected for this purpose consists in decomposing the problem into an investment master problem and an operating subproblem. This approach is attractive for two main reasons. First, the investment problem is a static one whereas the operating problem is dynamic so there is good reason to solve them separately. Second the operating subproblem has been studied quite extensively and referred to in many readily accessible publications whereas the master problem is far from well-known.

The numerical results reported here are highly conclusive, even though the number of sites was not very large and the subproblem was solved for a simple period of one year only. The short CPU time of 0.233 min is sufficiently encouraging to believe that this method could be applied with confidence to far bigger problems.

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